

北京大学物理学院“萃英”研究生学术论坛

Benford定理 在物理学中的应用

报告人：Lijing Shao (邵立晶)

指导老师：B.-Q. Ma (马伯强 教授)

小组研究简介

▶ **马伯强教授**小组现有成员约30人（教授1名，研究生19人，本科生10人），属于理论物理方向“人多势众”的“帮派”，涉足领域广泛

- 熵力=引力？

- 强子结构，实验与理论并进

- ▶ 光锥量子场论

- ▶ 轻子强子深度非弹散射，电子正电子湮灭过程中强子产生

- ▶ 微扰量子色动力学的应用性

- ▶ 原子核内的夸克效应，特殊的强子态

- ▶ ...

- 洛伦兹破坏的理论与唯象研究

- 夸克轻子统一性，额外维模型

- 宇宙学：宇宙高能射线等

- ...



马伯强教授





北京国家森林公园

2010-04-18

My Publication List

▶ 强子结构:

- ▶ L. Shao, Y.-J. Zhang, B.-Q. Ma, Phys. Lett. B 686 (2010) 136
- ▶ L. Shao, Y. Zhang, B.-Q. Ma, Chin. Phys. C (QNP09) in press
- ▶ Y. Zhang, L. Shao, B.-Q. Ma, Phys. Lett. B 671 (2009) 30
- ▶ Y. Zhang, L. Shao, B.-Q. Ma, Nucl. Phys. A 828 (2009) 390

▶ Benford 定理:

- ▶ L. Shao, B.-Q. Ma, Astropart. Phys. 33 (2010) 255
- ▶ L. Shao, B.-Q. Ma, Mod. Phys. Lett. A 24 (2009) 3275
- ▶ L. Shao, B.-Q. Ma, Physica A (2010) in press

▶ 洛伦兹破坏:

- ▶ L. Shao, Z. Xiao, B.-Q. Ma, Astropart. Phys. (2010) in press
- ▶ L. Shao, B.-Q. Ma, submitted

▶ 天文数据处理:

- ▶ L. Shao, J.-F. Liu, Y.-J. Yang, A. Kong, in preparation

Benford定理

在物理学中的应用

报告人: Lijing Shao (邵立晶)

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2010-04-23 @ 北京大学物理学院

报告提纲

- ▶ Benford定理简介
- ▶ Benford定理在物理学中的应用
 - 粒子物理中的Benford定理
 - 脉冲星——Benford理想数据库
 - Benford定理的一些性质
 - 从统计物理到Benford定理
- ▶ 报告总结
- ▶ 科研心得交流



引子：从物理常数说起



- ▶ $e = 1.60 \times 10^{-19} \text{ C}$
- ▶ $c = 3 \times 10^8 \text{ m s}^{-1}$
- ▶ $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
- ▶ $\hbar = 1.055 \times 10^{-34} \text{ J s}$
- ▶ $e = 2.718$
- ▶ $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$
- ▶ $m_{\text{Pl}} = 2.18 \times 10^{-8} \text{ kg}$
- ▶ $\mu_0 = 1.26 \times 10^{-6} \text{ N A}^{-2}$
- ▶ $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-2}$
- ▶ $m_p = 1.67 \times 10^{-27} \text{ kg}$
- ▶ $\pi = 3.1416$
- ▶ $\mu_B = 9.27 \times 10^{-24} \text{ J T}^{-1}$
- ▶

共12个常数：

- 5个1开头
- 2个2开头
- 2个3开头
- 1个6开头
- 1个8开头
- 1个9开头

| | | | |
|---------|---------|-------------|--------------|
| amu | G | λ_c | ϵ_0 |
| N_A | E_0 | m_d | h |
| μ_B | m_n | m_e | m_p |
| a_0 | μ_n | e | m_p |
| k | μ_0 | R | c |

Physics for Scientists and Engineers, 3rd ed. Serway (1990)

Burke & Kincanon, *Am. J. Phys.* 59 (1991) 952

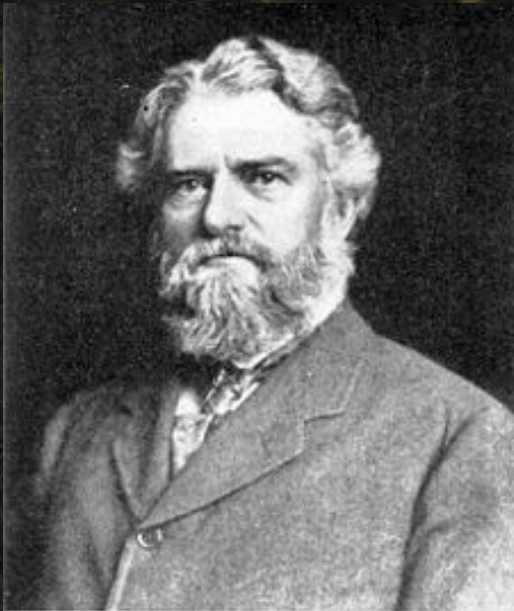
Table I. Distribution of first digits for 20 physical constants.

| k | mks system | | English system | | $\log(k + 1)/k$ |
|-----|------------|----|----------------|----|-----------------|
| | Frequency | % | Frequency | % | |
| 1 | 8 | 40 | 7 | 35 | 0.30 |
| 2 | 2 | 10 | 2 | 10 | 0.18 |
| 3 | 1 | 5 | 3 | 15 | 0.12 |
| 4 | 0 | 0 | 1 | 5 | 0.10 |
| 5 | 2 | 10 | 1 | 5 | 0.08 |
| 6 | 3 | 15 | 3 | 15 | 0.07 |
| 7 | 0 | 0 | 0 | 0 | 0.06 |
| 8 | 2 | 10 | 1 | 5 | 0.05 |
| 9 | 2 | 10 | 2 | 10 | 0.05 |

物理常数和 Benford 定理

Newcomb与对数表

- ▶ 1881年，Newcomb发现对数表前面的书页脏得比后面的书页快，2页的短文
- ▶ [Newcomb, Am. J. Math. 4 (1881) 39]



- ▶ **Simon Newcomb (1835 – 1909)**
- ▶ 美籍加拿大人，天文学家，数学家。
- ▶ 主要贡献有：天文计时，经济学、统计学写作，科幻小说

Benford与对数表

- ▶ 1938年，Benford，对数表 (Again!)
- ▶ [Benford, Proc. Am. Phil. Soc. 78 (1938) 551]
- ▶ 竭尽所能，花两年时间统计：报纸上的数字，机械制作中的尺寸规范，分子原子的重量，湖泊的面积，河流的长度...

▶ Frank Benford (1883 – 1948)

- ▶ 美国电气工程师，物理学家
- ▶ 密西根大学，通用电气公司
- ▶ 发明了一种测量玻璃折射率的新方法，发表了109篇光学与数学方面的论文，申请20项光学专利



Benford 定理



- ▶ Benford's law, 又称为 the first digit law, the significant digit law 等

$$P(k) = \log_{10} \left(1 + \frac{1}{k} \right), \quad k = 1, 2, \dots, 9$$

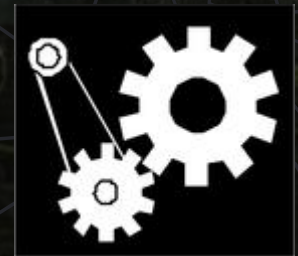
- ▶ 首 n -位数的分布

$$P_{\text{Ben}}(d_1, d_2, \dots, d_n) = \log_{10} \left[1 + \left(\sum_{i=1}^n d_i \times 10^{n-i} \right)^{-1} \right]$$

- ▶ 如, 前两位为15的概率为 $\log_{10}(1+1/15)$

Benford 定理的领域

- ▶ 报纸上的数字，分子原子的重量，湖泊的面积，河流的长度... Benford 1938
- ▶ 物理学常数 Burke & Kincanon 1991
- ▶ α 衰变半衰期 Buck et al. 1993, Ni & Ren 2008
- ▶ 存活率 Leemis et al. 2000
- ▶ 复杂原子跃迁中的电偶极能谱 Pain 2008
- ▶ 股票指数 Dorogovtsev et al. 2006
- ▶ eBay 交易额 Giles 2007
- ▶ 个人计算机中的文件大小 Torres et al. 2007
- ▶ 动力学系统 Tolle et al. 2000, Snyder et al. 2001, Berger et al. 2005
- ▶ ...



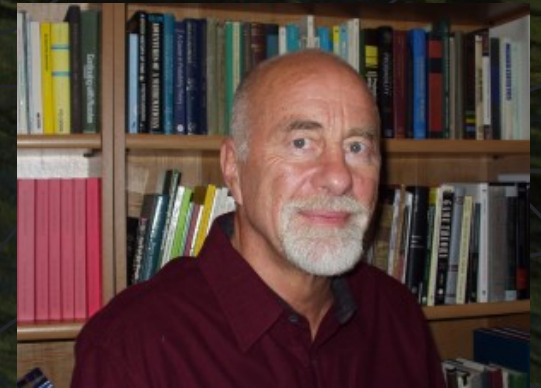


一个未解的“幽灵”之题

- ▶ 有些数据很好地符合Benford定理，而有些却完全不符合
- ▶ 如：门牌号，中奖号等
- ▶ 而且，我们无法先验地得知，一个数据集是否符合Benford定理；只有在详细统计后，才能下结论
- ▶ 如何判定Benford分布？也许涉及到自然界深层的规律？

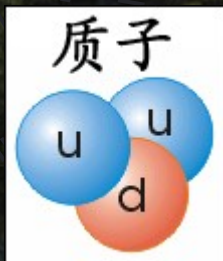
Benford 定理的应用

- ▶ 财务与会计学中的欺骗 Nigrini 1996, Nigrini 1999, Rose & Rose 2003
- ▶ 临床测试中的伪造 Al-Marzouki et al. 2005
- ▶ 选举作假 Torres et al. 2007
- ▶ 科学实验数据伪造 Diekmann 2007
- ▶ 提高计算机性能 Barlow & Bareiss 1985
- ▶ 减小计算机存储空间 Schatte 1988
- ▶ 图片处理 Jolion 2001, Fu et al. 2007
- ▶ ...
- ▶ T.P. Hill 的课堂实验
 - 新墨西哥大学教授



Benford 定理与粒子物理

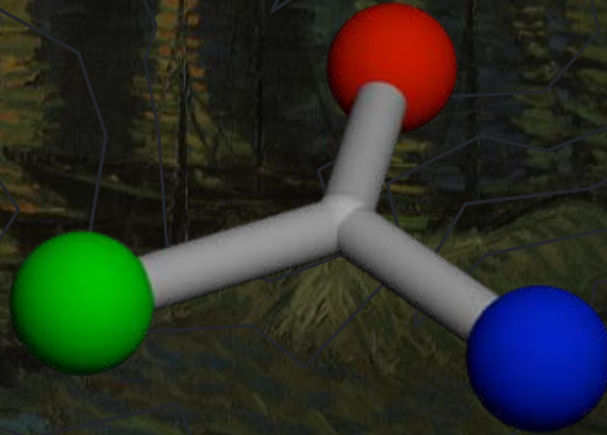
- ▶ Benford定理存在于广泛的领域，而在粒子物理领域却尚无人问津；那么，它在最基本的粒子领域是否仍旧成立呢？
- ▶ 我们首次发现，粒子的宽度 Γ 惊人地符合 Benford 分布
- ▶ Particle Data Group, Phys. Lett. B 667, 1 (2008)

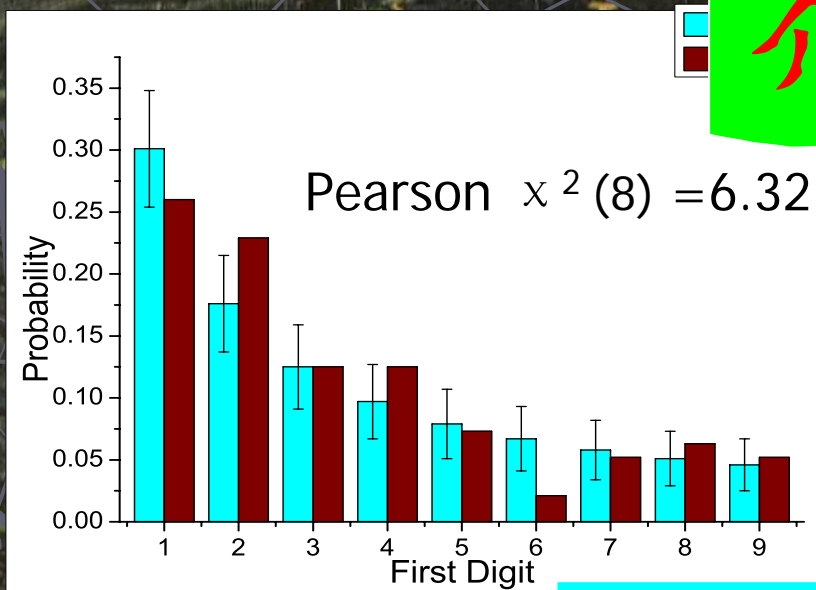
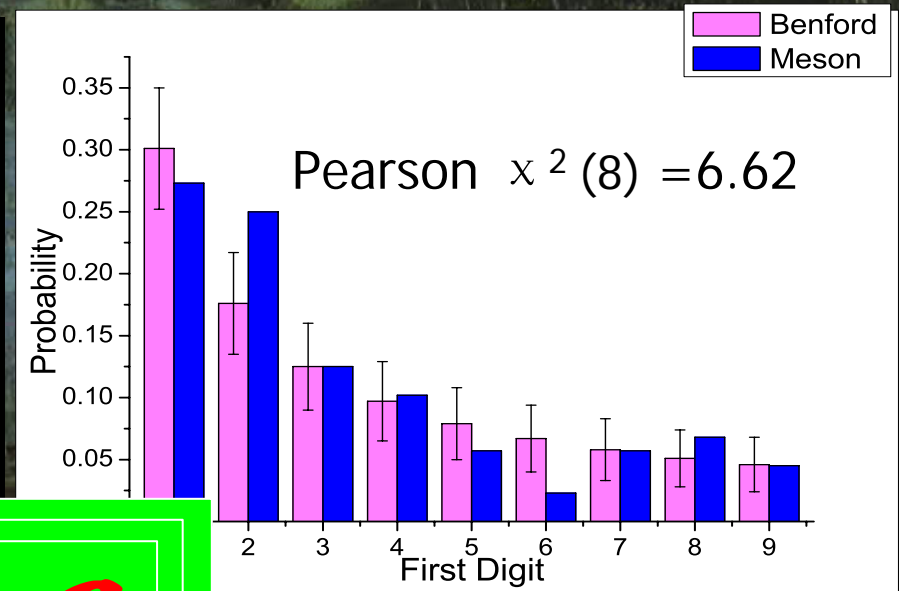
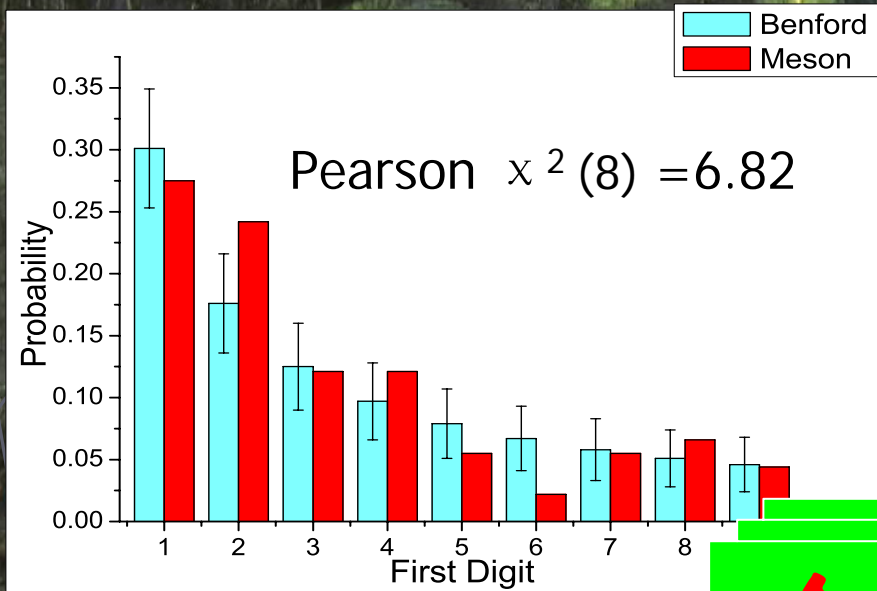


L. Shao & B.-Q. Ma, Mod. Phys. Lett. A 40 (2009) 3275

强子物理简介 (partial)

- ▶ 介子：正夸克+反夸克
- ▶ 重子：三个夸克
- ▶ 强子 = 介子+重子





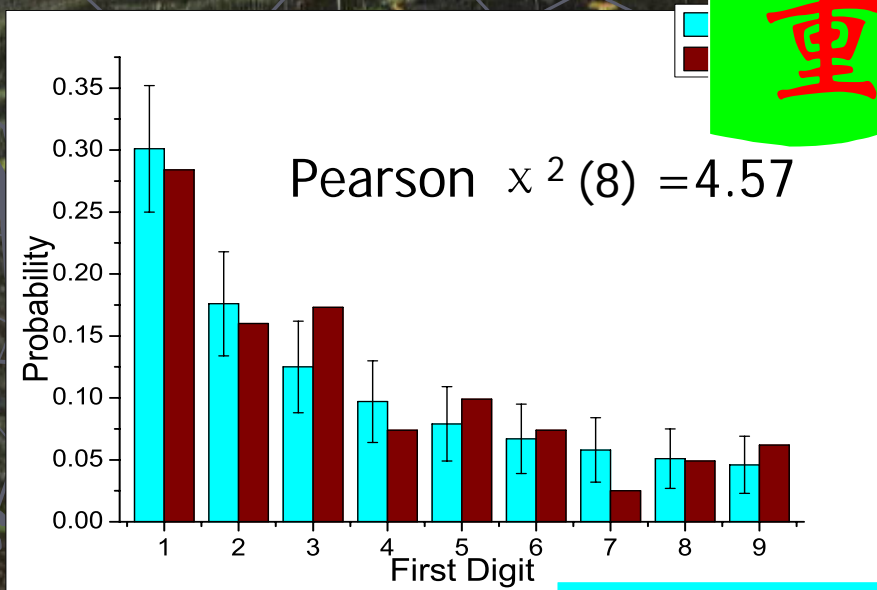
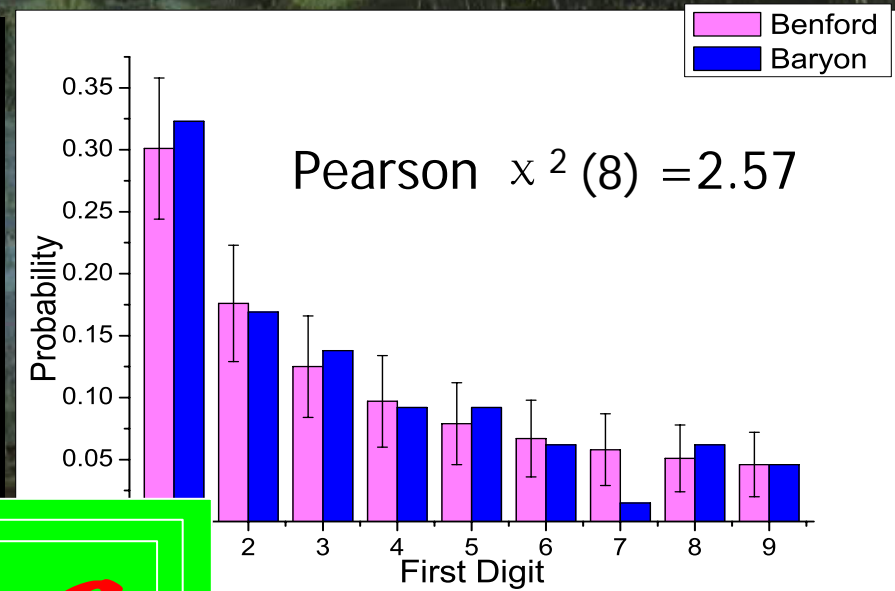
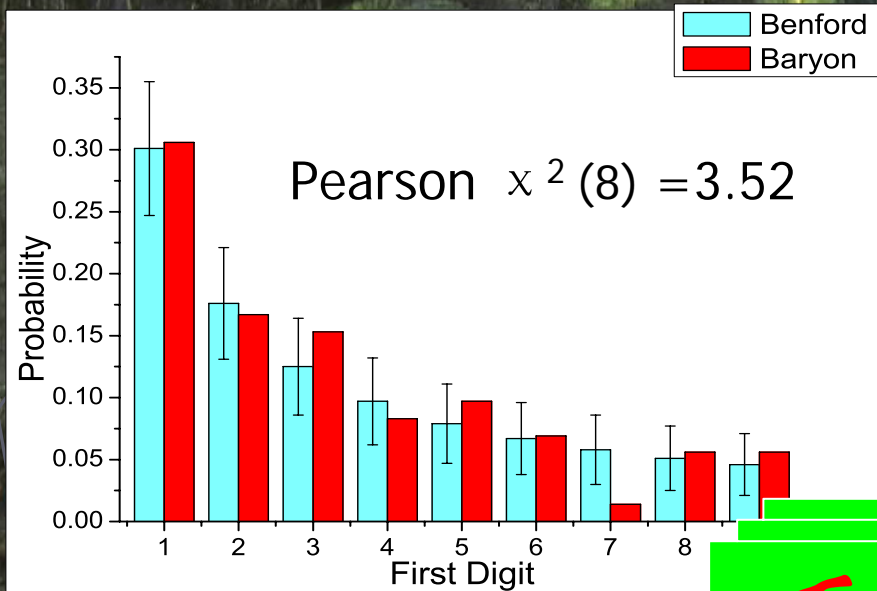
介子

$$N_{\text{Ben}} = N \log_{10}(1 + 1/k)$$

$$\Delta N = \sqrt{NP(k)(1 - P(k))}$$

$$\chi^2(n - 1) = \sum_{i=1}^n \frac{(N_{\text{Obs}} - N_{\text{Ben}})^2}{N_{\text{Ben}}}$$

【CL95%】 $\chi^2(8) = 15.507$



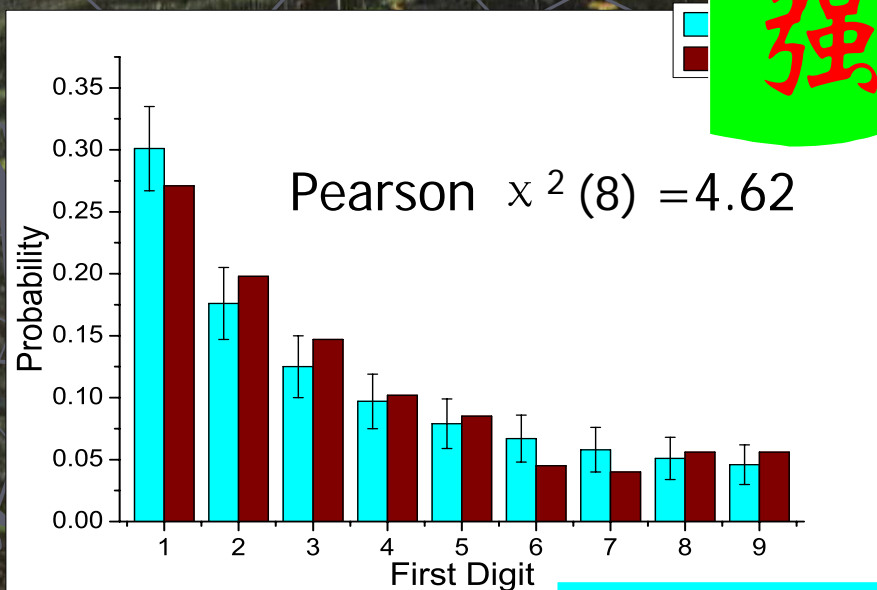
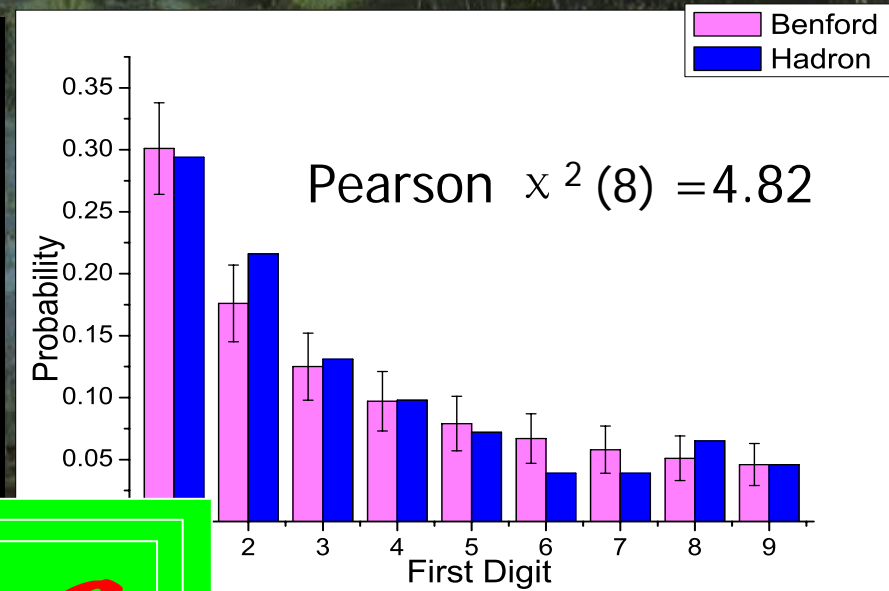
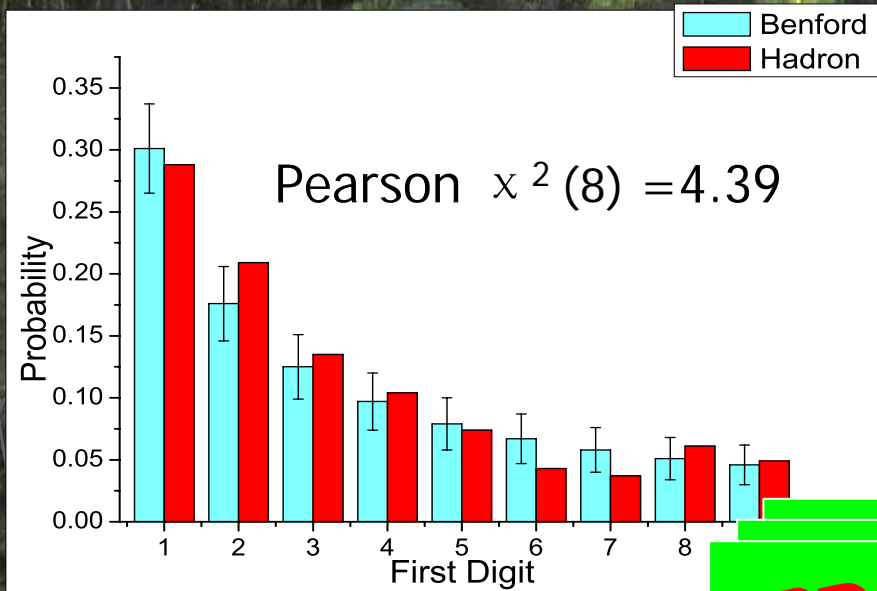
重子

$$N_{\text{Ben}} = N \log_{10}(1 + 1/k)$$

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强子

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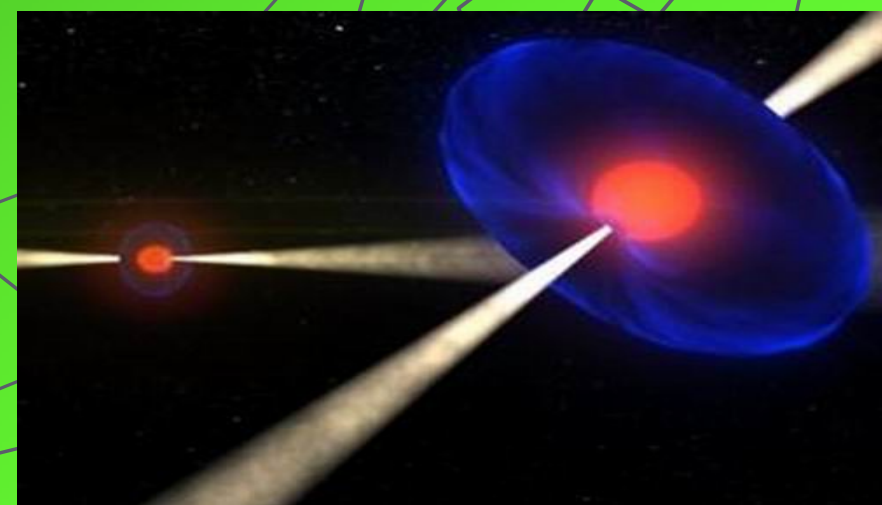
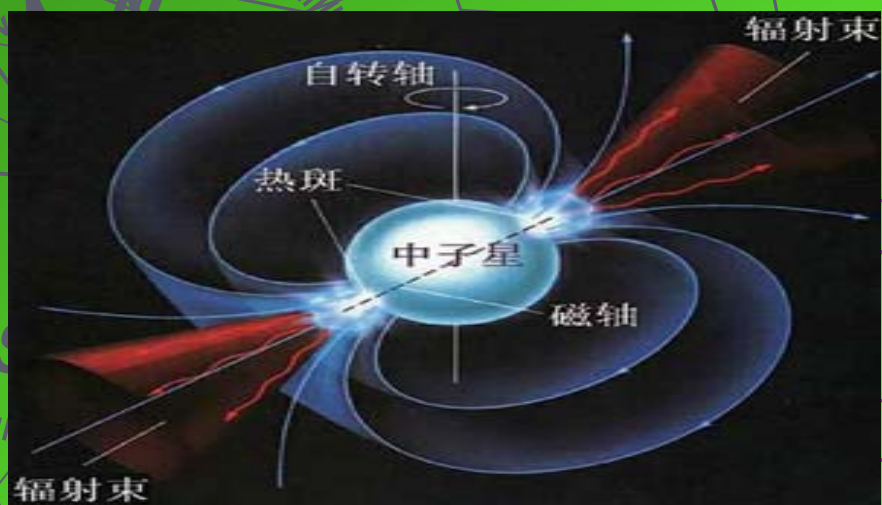
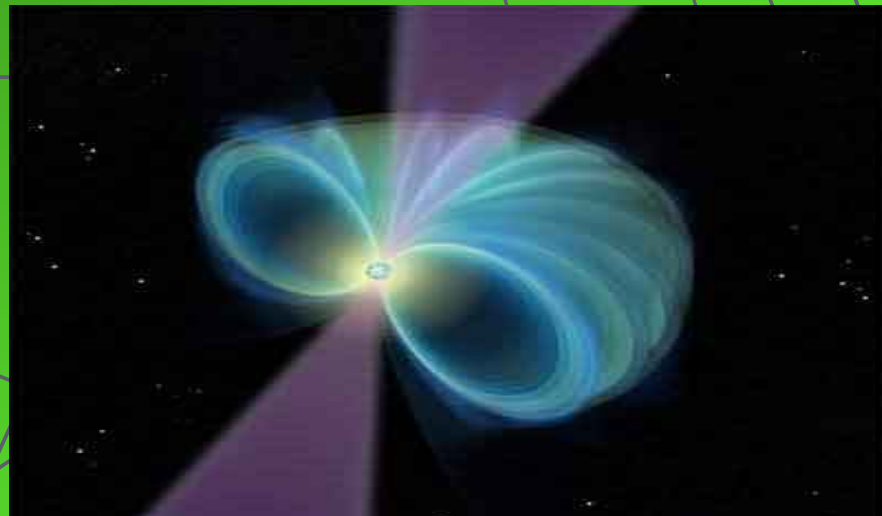
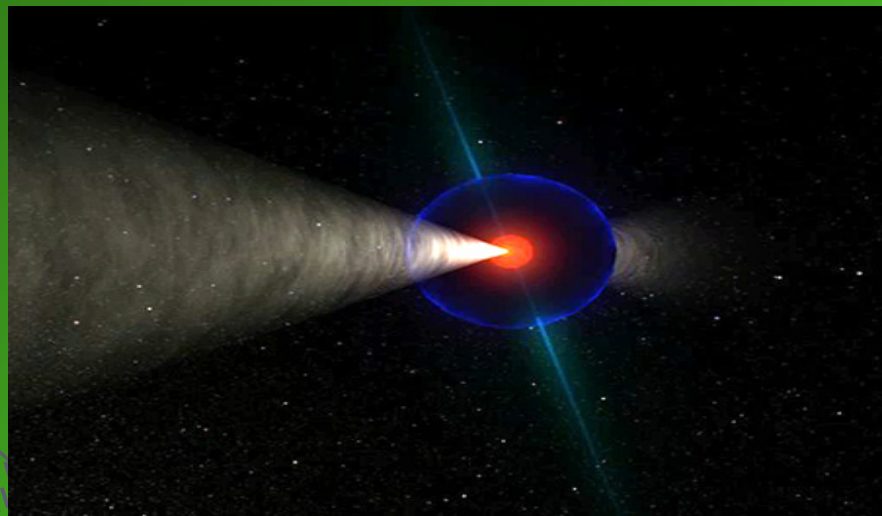
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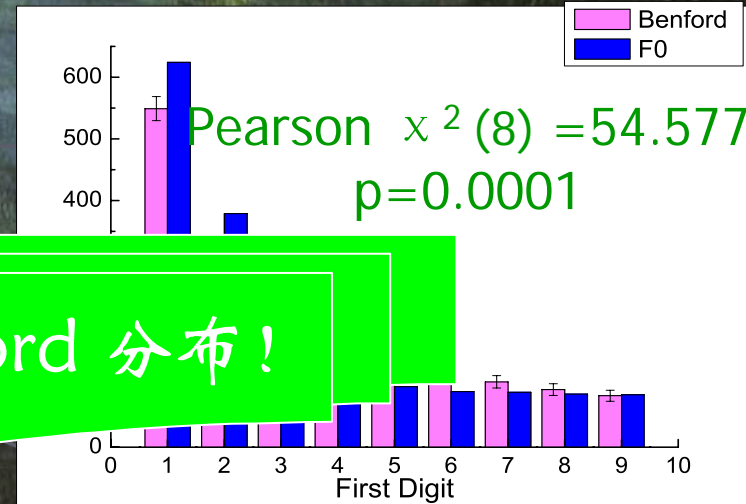
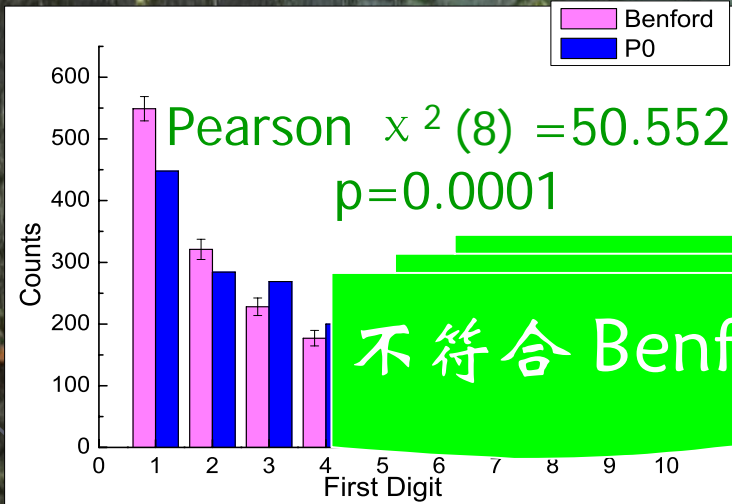
Benford 定理与脉冲星

- ▶ 既然 Benford 定理在最微观的领域上也成立；那么，在天体尺度上又如何呢？
- ▶ 我们系统地研究了脉冲星的各个物理量，发现了很多有意思的现象；并指出，脉冲星数据库可作为学习、检验与研究 Benford 定理的**理想场所**
- ▶ Australia Telescope National Facility (ATNF) pulsar catalogue (<http://www.atnf.csiro.au/research/pulsar/psrcat>)

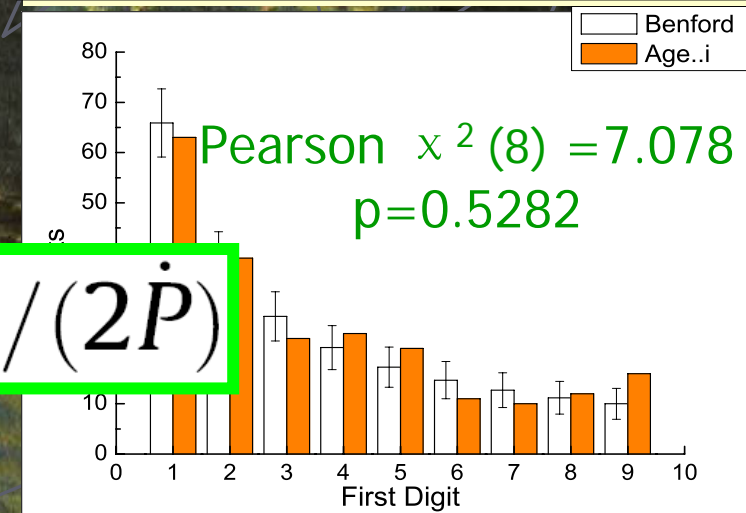
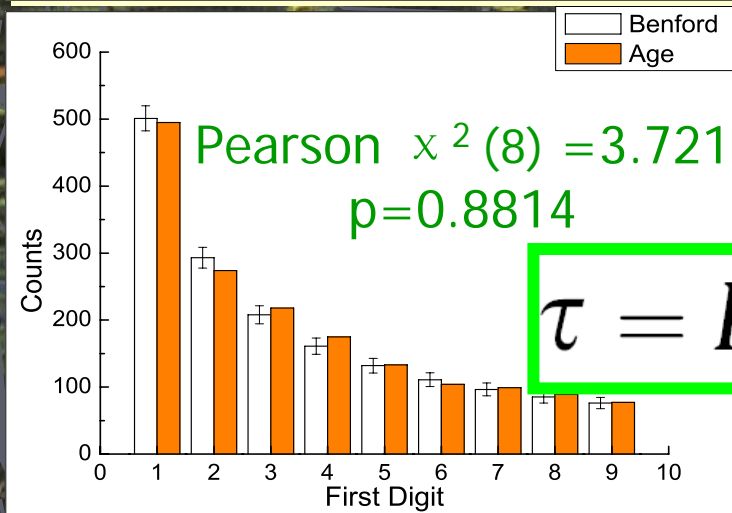
脉冲星——天空中的灯塔



脉冲星的周期、自转频率和年龄



不符合 Benford 分布!



$$\tau = P / (2\dot{P})$$

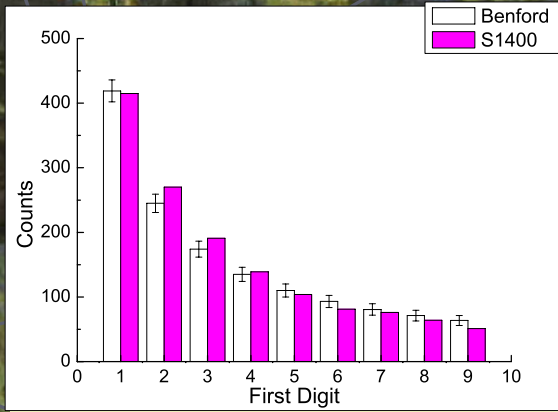
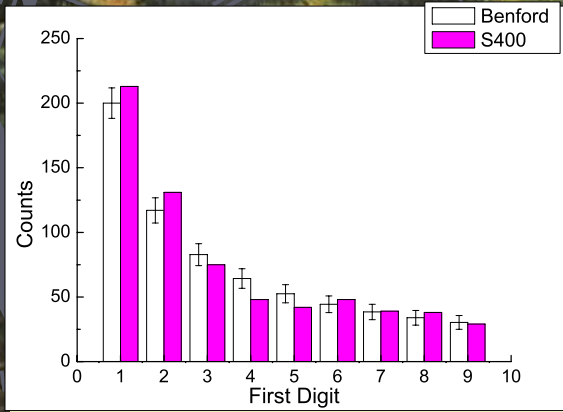
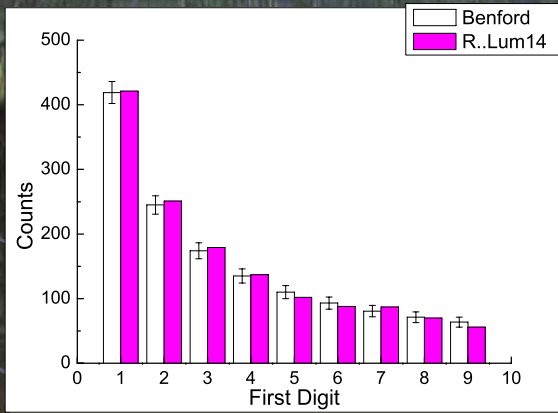
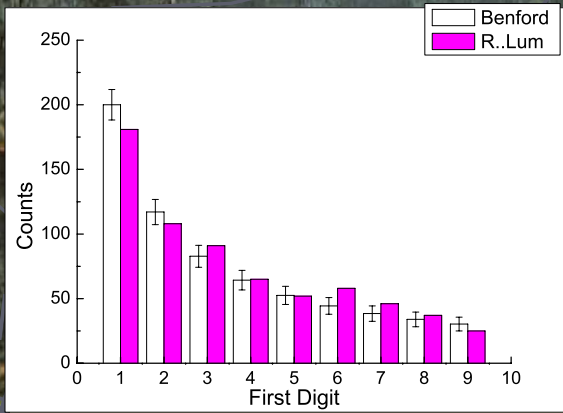
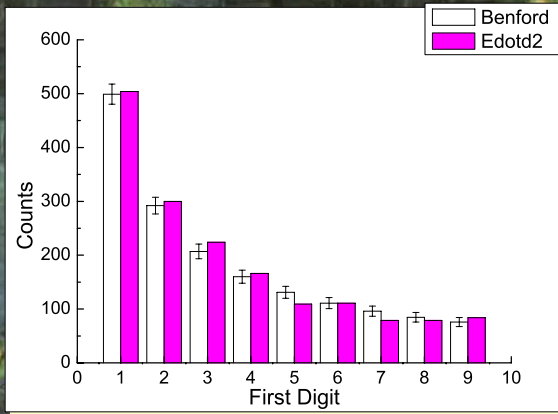
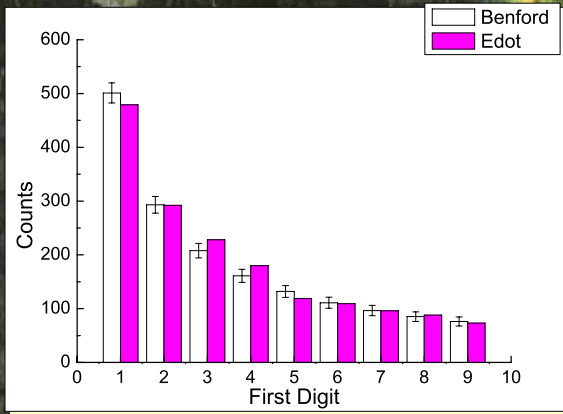
随机相乘与Benford定理

- ▶ 选取一组数，如1-500，随机抽取四个，相乘，得到新数，加入原数据集；重复进行多次
- ▶ 新的数据集符合Benford分布！
- ▶ Torres et al. (2007) *Eur. J. Phys.*

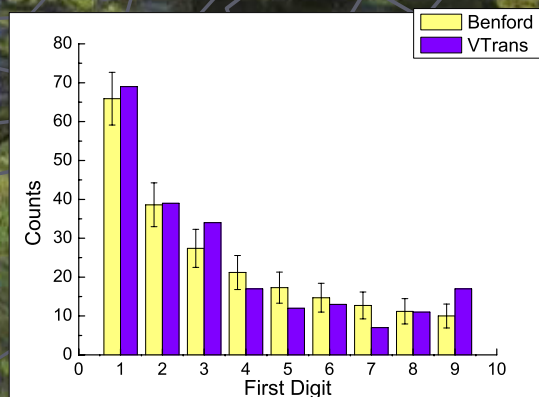
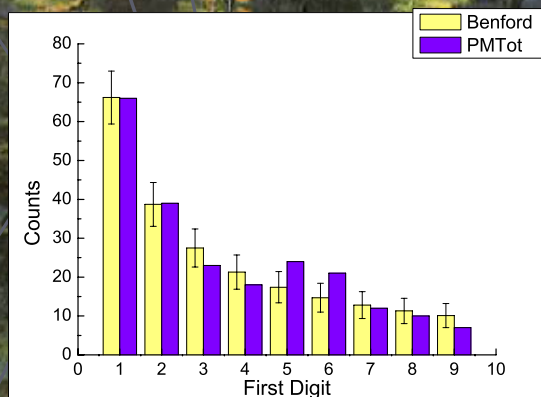
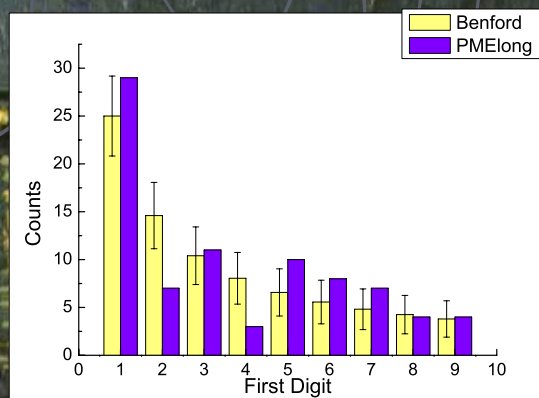
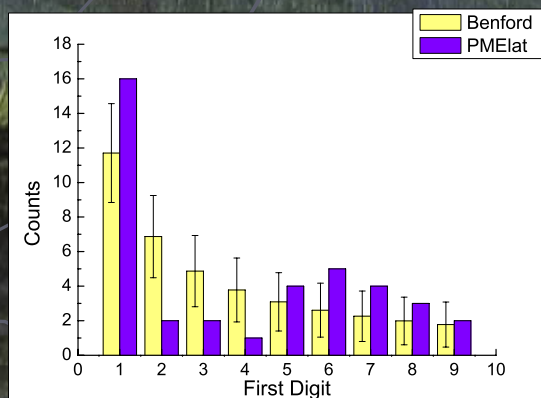
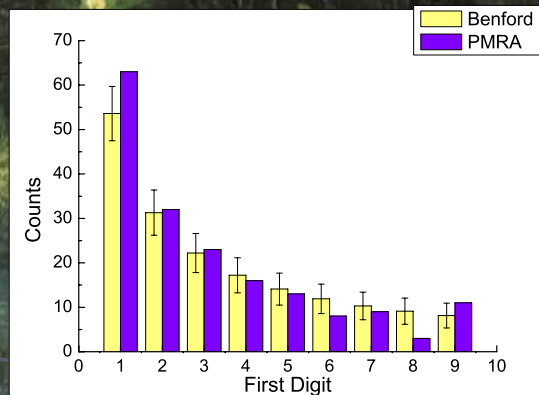
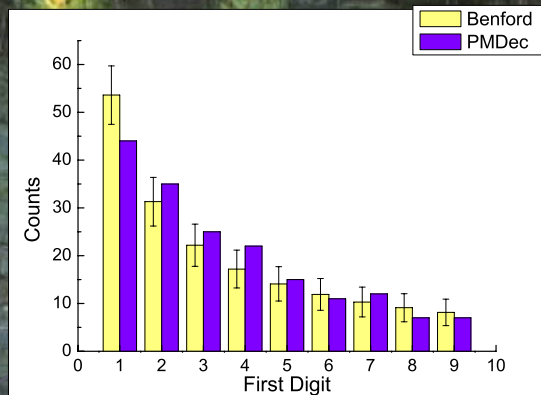
$$\tau = P / (2\dot{P})$$

- ▶ $(P: \text{non-Benford}) \div (P\dot{:} \text{Benford})$
= $(\tau: \text{Benford-数据集})$

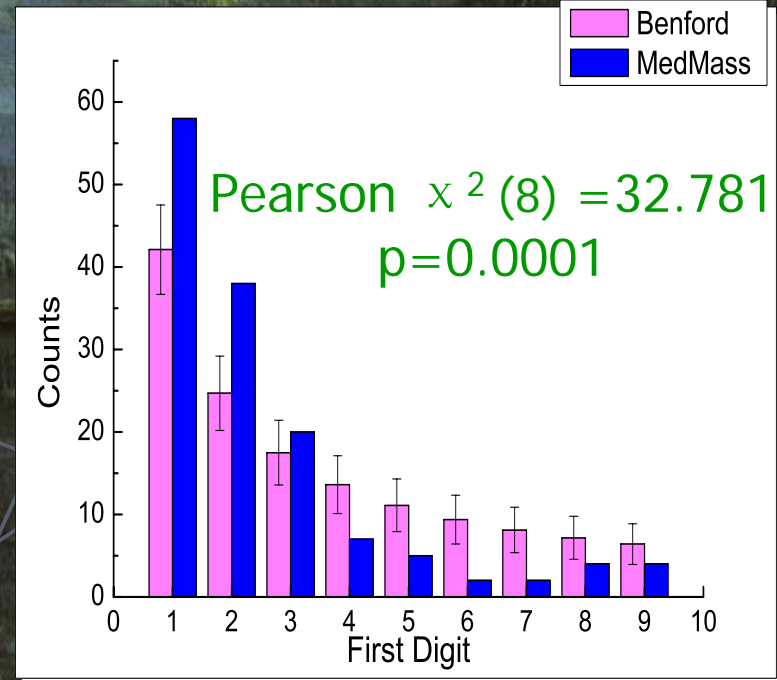
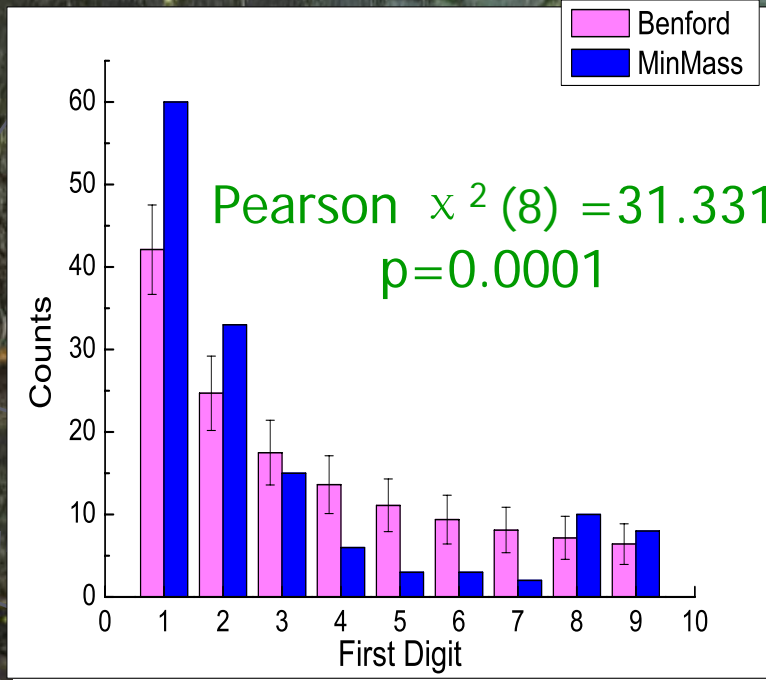
脉冲星的功率物理量



脉冲星的运动物理量

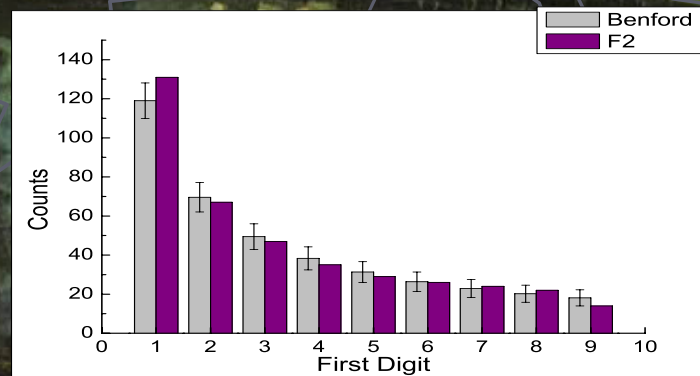
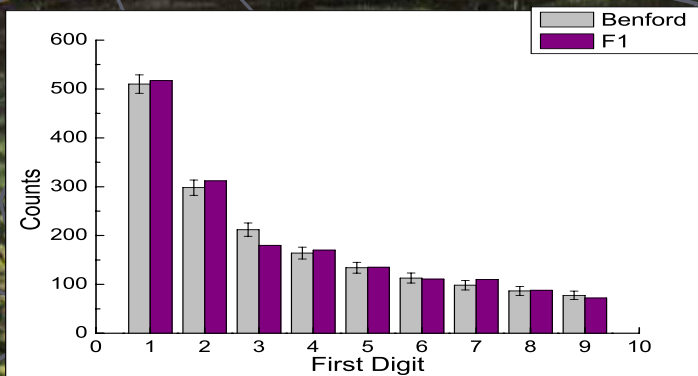
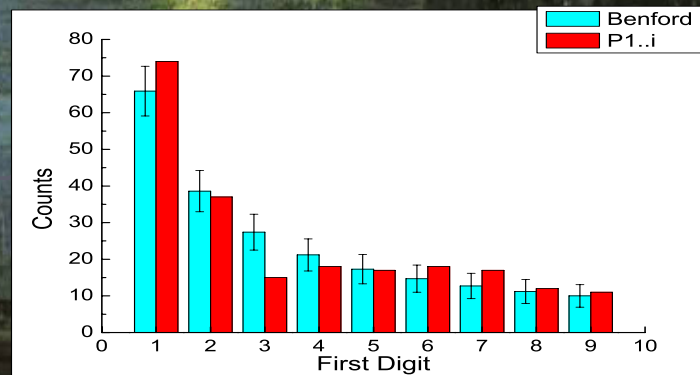
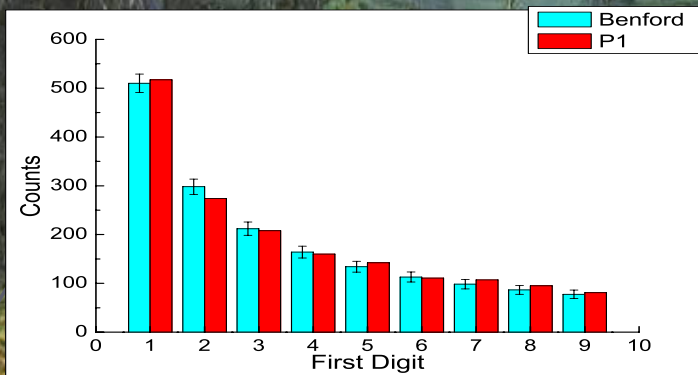


脉冲星的伴星质量



有人为因素与限制的量不符合 Benford 定理!

脉冲星的周期、自转频率的导数



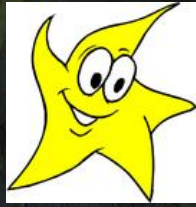
Benford数据集的时间导数也符合Benford定理!

正负数据集分别符合Benford定理!

关于 Benford 定理的进一步讨论

- ▶ 对数空间
- ▶ 标度不变性
- ▶ 整数幂次不变性
- ▶ 基数不变性
- ▶ 相乘过程 (推广的 Benford 定理)
- ▶ 尾数分布
- ▶ ...





对数空间

- ▶ Newcomb指出，把数据集写成 $\{10^{n+f}\}$ ，如果 f 在 $[0,1)$ 上均匀分布，那么，它则符合 Benford 分布
- ▶ Newcomb (1881) Am. J. Math.
- ▶ **尾数** (mantissa)：从实数 a 到实数 m 的映射， $a = m \times 10^n \rightarrow m \in [0.1,1)$
- ▶ 如：314.5 \rightarrow 0.3145，0.063 \rightarrow 0.63，3223 \rightarrow 0.3223

标度不变性

- ▶ 如果把某个Benford数据集统一换个标度(乘以一个非零常数), 那么它仍旧符合Benford分布
- ▶ Pinkham (1961) Ann. Math. Statist.

$$\log_{10} N' = \log_{10}(C \times N) = \log_{10} C + \log_{10} N$$

- ▶ 不同单位制对 Benford 分布没有影响
- ▶ Benford定理是数学上唯一的具有标度不变性的数位(digital)定理
- ▶ Berger, Hill & Morrison (2008) J. Theor. Probab.

整数幂次不变性

- ▶ 非零整数次幂也不改变 Benford 分布
- ▶ $\log_{10}N' = \log_{10}N^n = n \cdot \log_{10}N$
- ▶ Shao & Ma (2009) Mod. Phys. Lett. A



- ▶ 所以，由强子宽度与寿命 $\Gamma \tau = \hbar$ ，可知，强子寿命也符合 Benford 定理
- ▶ Shao & Ma (2009) Mod. Phys. Lett. A

- ▶ 脉冲星的周期与自转频率
- ▶ Shao & Ma (2009) Astropart. Phys.

基数不变性

- ▶ Benford定理与所选用的数位进制也没有关系：二进制，八进制，十进制，百进制...
- ▶ $P(d) = \log_b(1+1/d)$, $d=1,2,\dots,b-1$
- ▶ Raimi 1969a, Raimi 1969b, Raimi 1976, Raimi 1985
- ▶ Hill 1995a, Hill 1995b, Hill 1995c, Hill 1998, Hill et al. 2005
- ▶ 首n-位数的Benford定理，可理解为100进制(前两位数字)、1000进制(前三位数字)下的结果
- ▶ Hill 1995a,b,c: 标度不变性意味着基数不变性，基数不变性意味着Benford定理

相乘过程

- ▶ $m(t + \delta t) = r(t)m(t)$
- ▶ 如：股票价格等
- ▶ $\log_{10} m(t + \delta t) = \log_{10} r(t) + \log_{10} m(t)$
- ▶ 对数空间中的布朗过程
- ▶ 大数定理：方差 $\rightarrow \infty$ ， $\log_{10} m(t) \rightarrow$ 均匀分布
- ▶ Pietronero et al. (2001) *Physica A*



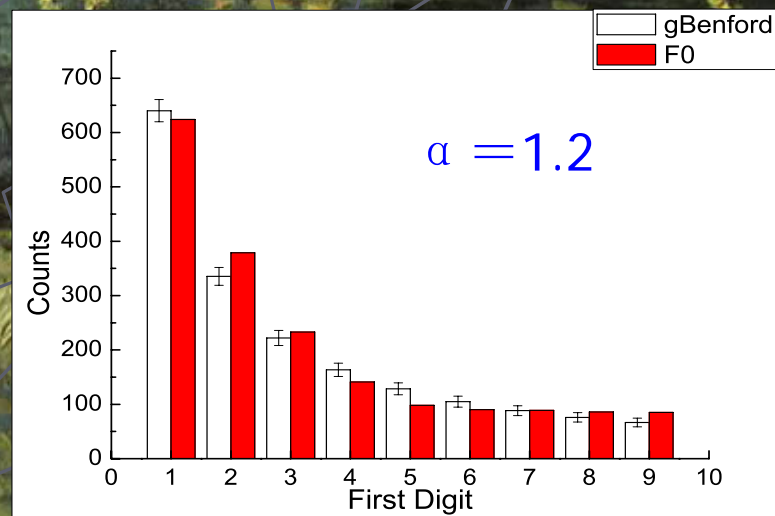
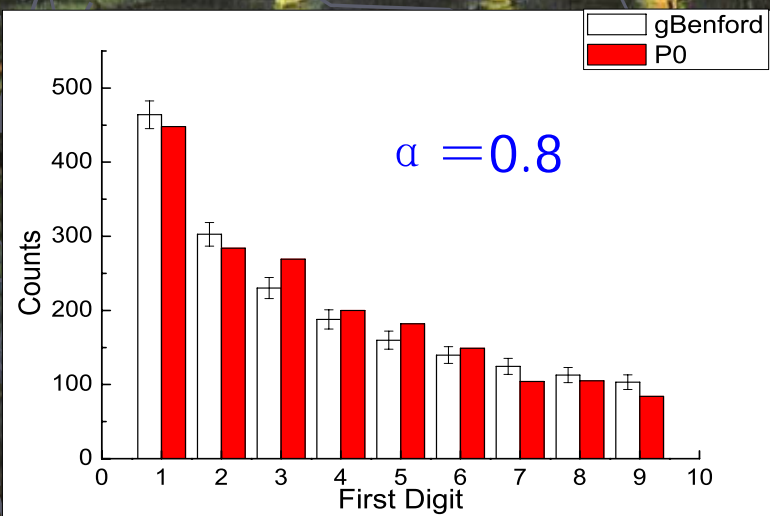
$$P(k) = \frac{\int_k^{k+1} d \log m(t)}{\int_1^{10} d \log m(t)} = \frac{\int_k^{k+1} m^{-1} dm}{\int_1^{10} m^{-1} dm} = \log_{10} \left(1 + \frac{1}{k} \right)$$

推广的 Benford 定理

$$P_{\alpha}(k) = C \int_k^{k+1} m^{-\alpha} dm$$

Pietronero et al. (2001) Physica A

$\alpha = 0$: 均匀分布; $\alpha = 1$: Benford 分布

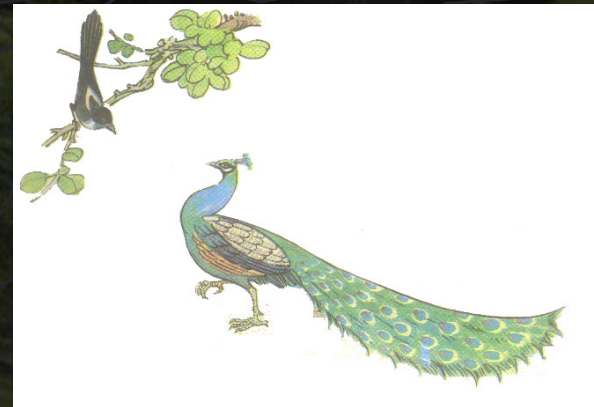


尾数分布

- ▶ 首n-位数的Benford分布必然意味着尾数的 $1/m$ 的分布；而尾数的 $1/m$ 的分布又必然意味着n-位的Benford分布

$$f(m) = \frac{1}{\ln 10} \cdot \frac{1}{m}, m \in [0.1, 1)$$

- ▶ Shao & Ma, Physica A (2010) in press



Benford定理与统计物理

- ▶ 既然Benford定理如此广泛地适用于各领域的现象，那么，它是否与正则的统计物理有着联系？
- ▶ 玻尔兹曼-吉布斯(BG)分布，费米-狄拉克(FD)分布，玻色-爱因斯坦(BE)分布
- ▶ Shao & Ma, Physica A (2010) in press

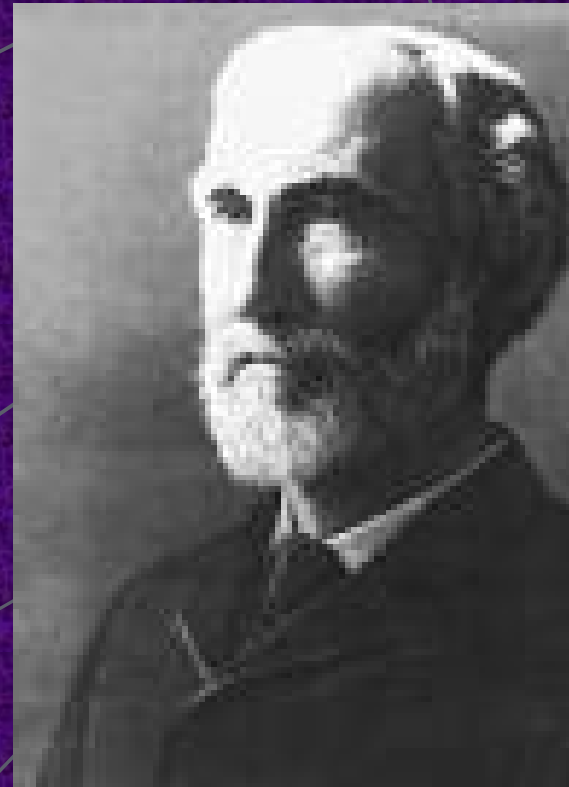
数学准备

$$P(d) = \sum_{n=-\infty}^{\infty} \int_{d \cdot 10^n}^{(d+1) \cdot 10^n} f(E) dE$$

- ▶ $f(E)$: 概率密度; E : 随机变量
- ▶ Engel & Leuenberger (2003) Stat. Probabil. Lett.
- ▶ Shao & Ma, Physica A (2010) in press

快乐数学

玻尔兹曼-吉布斯统计



Boltzmann, Wien. Ber. 76 (1877) 373-435.

Gibbs, *Elementary Principles in Statistical Mechanics*, Dover, New York, 1901.

玻尔兹曼分布与 Benford 定理

$$f_{BG}(E) = \beta e^{-\beta E} \quad P_{BG}(d; \beta) = \sum_{n=-\infty}^{\infty} [e^{-\beta d \cdot 10^n} - e^{-\beta(d+1) \cdot 10^n}]$$

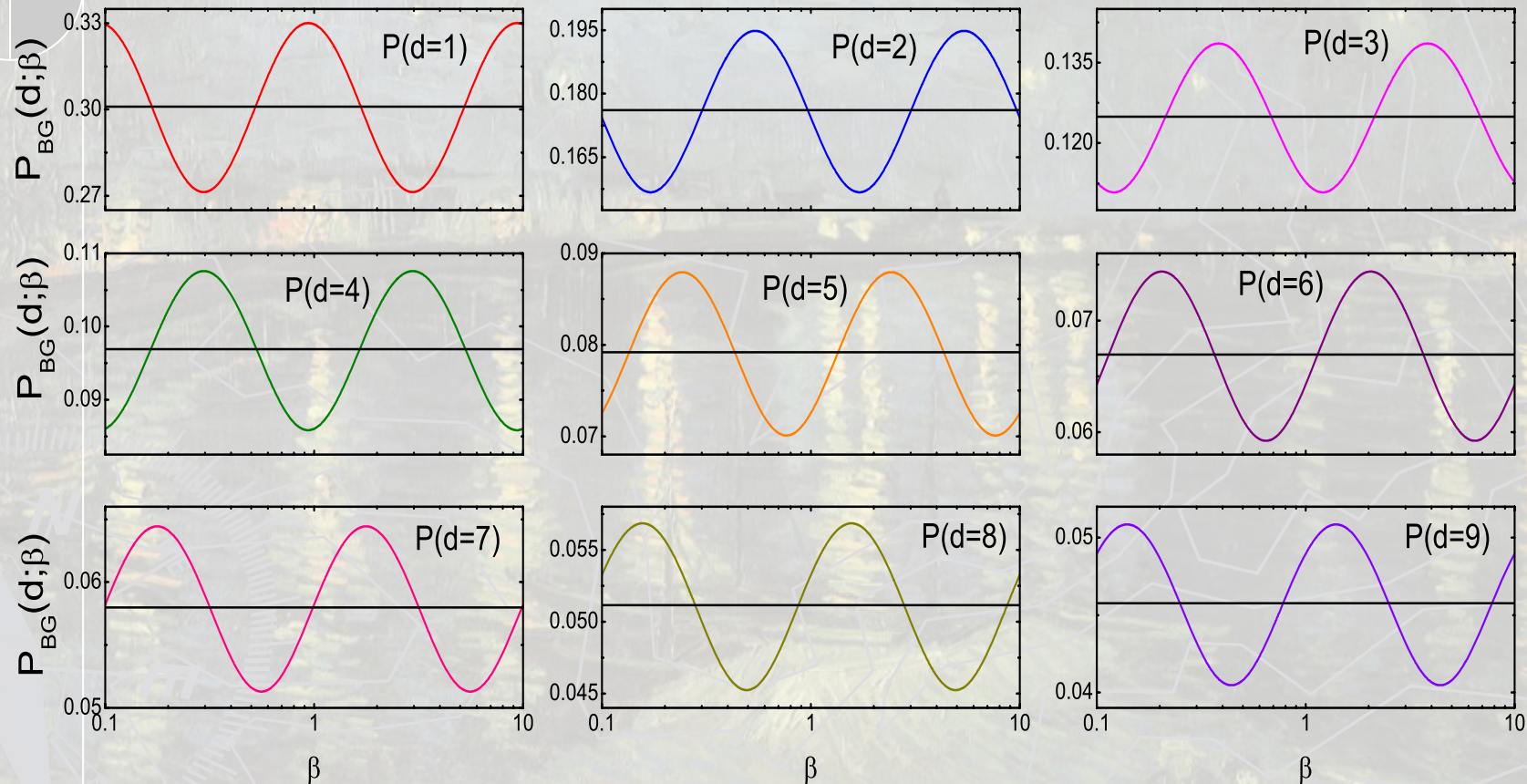
Engel & Leuenberger (2003) Stat. Probabil. Lett.
Shao & Ma, Physica A (2010) in press

$$P_{BG}(d; \beta) = P_{BG}(d; 10\beta)$$

$$P_{BG}^*(d; \alpha) = P_{BG}(d; \beta = 10^\alpha) \quad P_{BG}^*(d; \alpha) = P_{BG}^*(d; \alpha + 1)$$

$$\int_0^1 P_{BG}^*(d; \alpha) d\alpha = \log_{10}\left(1 + \frac{1}{d}\right) \equiv P_{\text{Ben}}(d)$$

玻尔兹曼分布与 Benford 定理



$$f_{BG}(E) = \beta e^{-\beta E}$$

L. Shao & B.-Q. Ma, Physica A (2010) in press

费米-狄拉克统计



E. Fermi, Rend. Lincei 3 (1926) 145-149.

P.A.M. Dirac, Proc. R. Soc. A 112 (1926) 661-677.

费米-狄拉克分布与Benford定理

$$f_{FD}(E) = \frac{\beta}{\ln 2} \frac{1}{e^{\beta E} + 1}$$

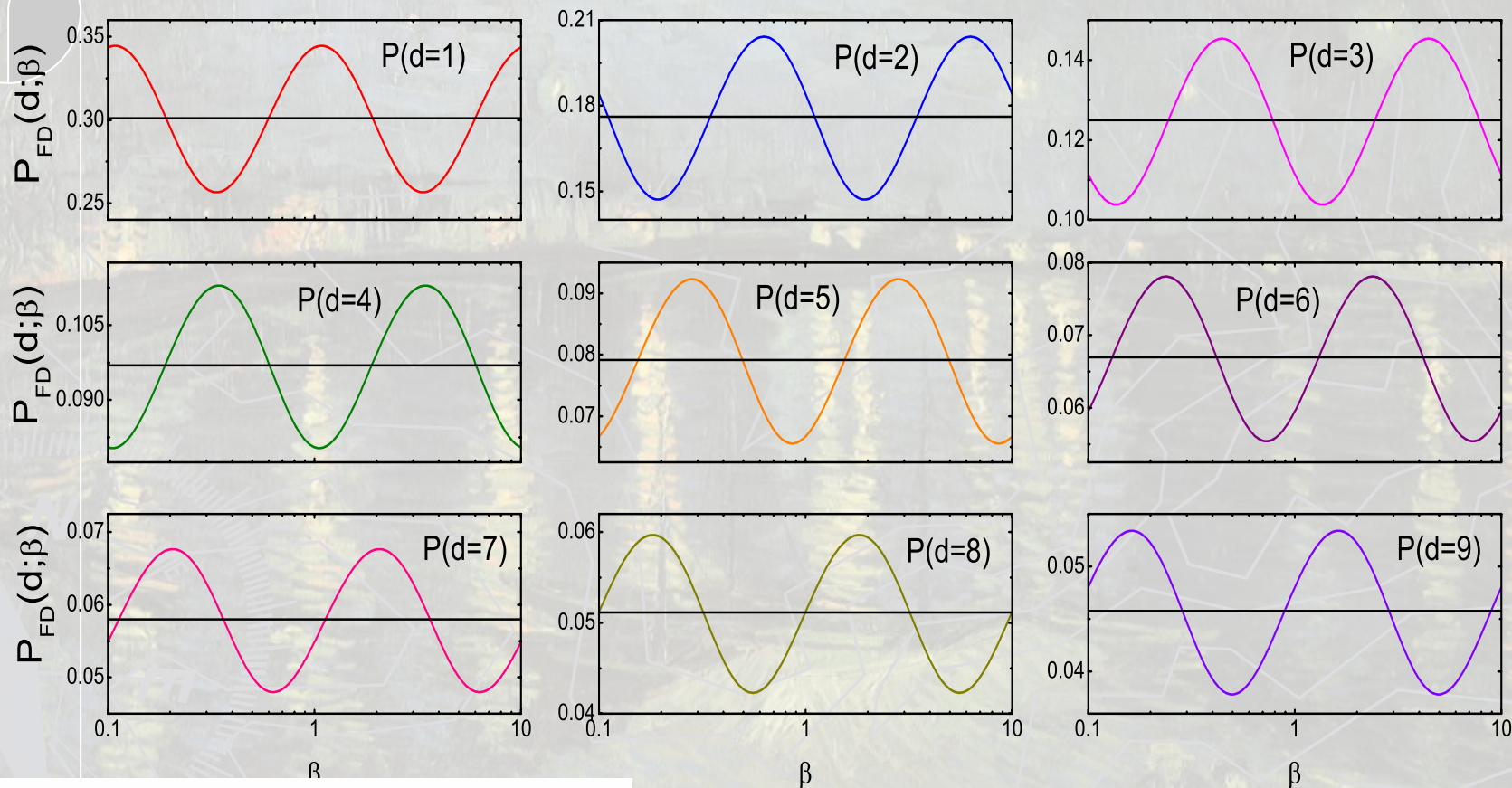
$$P_{FD}(d; \beta) = \sum_{n=-\infty}^{\infty} \frac{1}{\ln 2} \left\{ \beta \cdot 10^n + \ln \left[\frac{e^{\beta d \cdot 10^n} + 1}{e^{\beta(d+1) \cdot 10^n} + 1} \right] \right\}$$

L. Shao & B.-Q. Ma, Physica A (2010) in press

$$P_{FD}^*(d; \alpha) = P_{FD}(d; \beta = 10^\alpha)$$

$$\int_0^1 P_{FD}^*(d; \alpha) d\alpha = \log_{10} \left(1 + \frac{1}{d} \right) \equiv P_{\text{Ben}}(d)$$

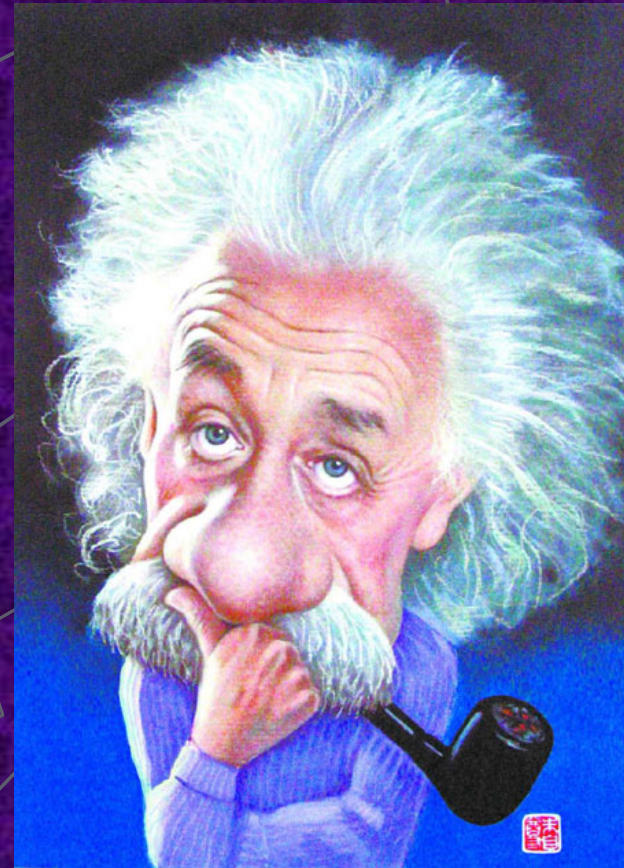
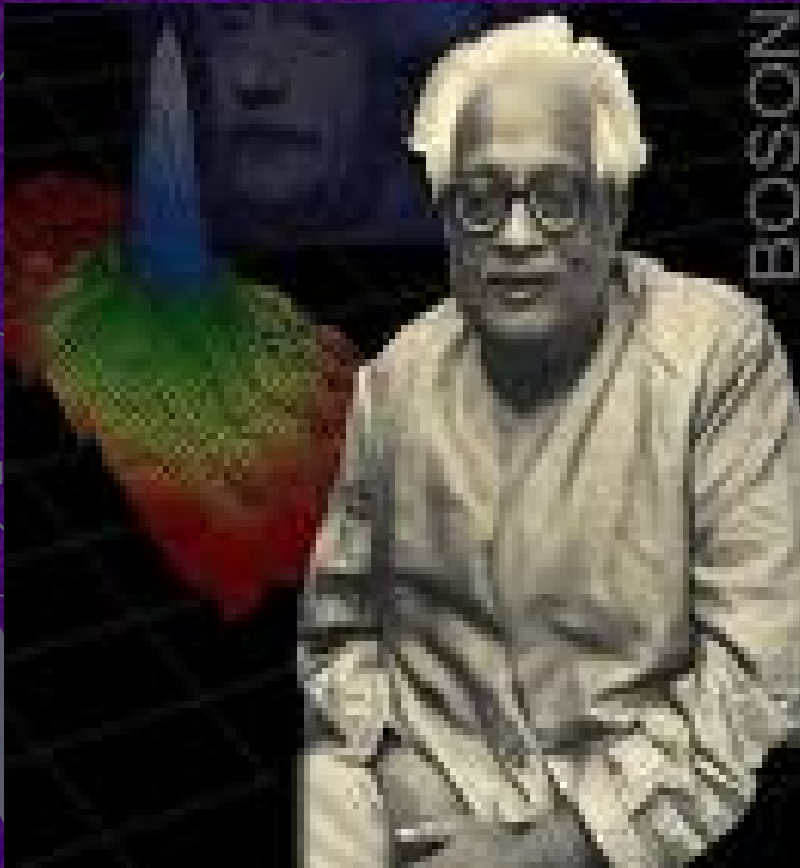
费米狄-拉克分布与Benford定理



$$f_{FD}(E) = \frac{\beta}{\ln 2} \frac{1}{e^{\beta E} + 1}$$

L. Shao & B.-Q. Ma, Physica A (2010) in press

玻色-爱因斯坦统计



S.N. Bose, *Z. Phys.* 26 (1924) 178-181.

A. Einstein, *Sitz. Ber. Preuss. Akad. Wiss.* 22 (1924) 261-267.

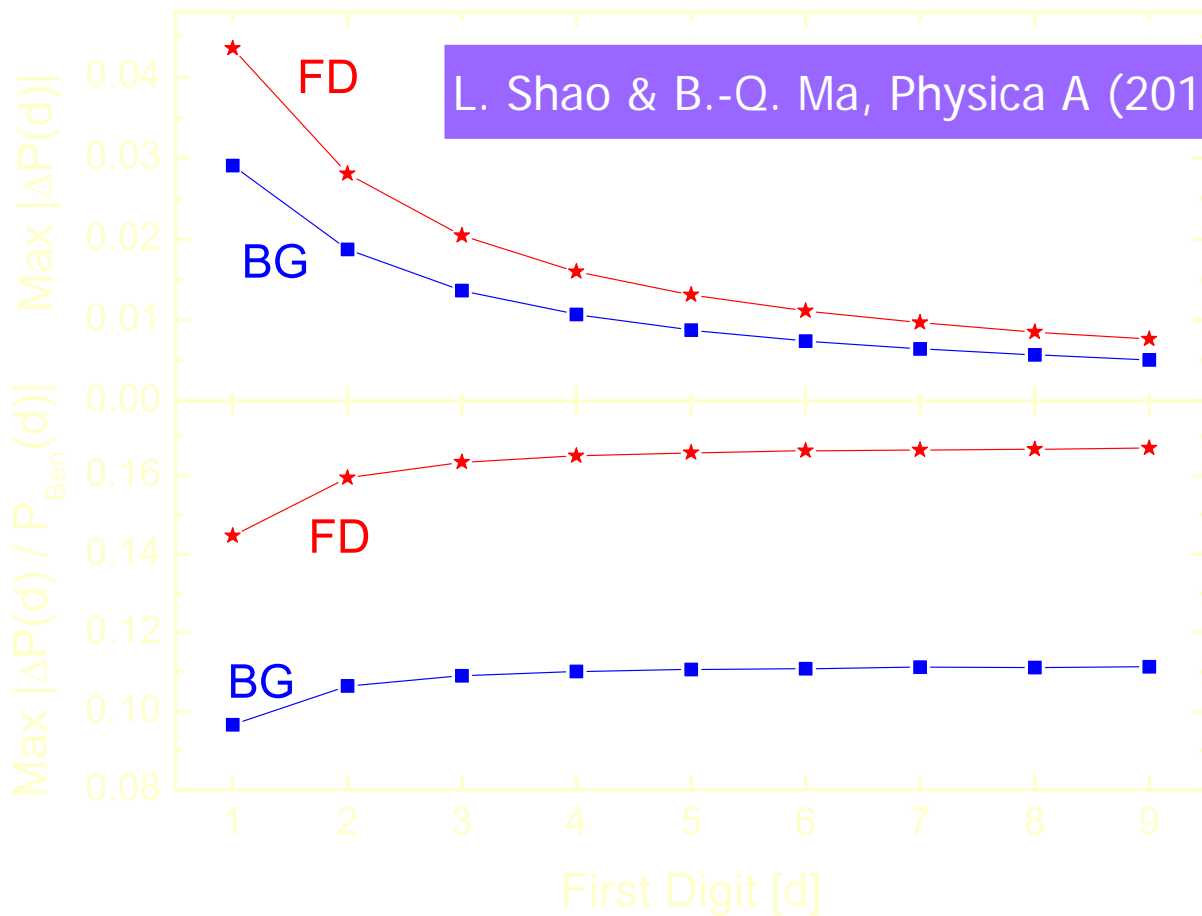
玻色-爱因斯坦分布与Benford定理

$$f_{BE}(E) \propto \frac{1}{e^{\beta E} - 1}$$

- ▶ $f_{BE} \rightarrow 1/E, E \rightarrow 0$
- ▶ 数位的全部贡献都由 $E \rightarrow 0$ 部分提供，根据尾数分布的讨论，玻色-爱因斯坦分布**精确地**满足Benford定理！

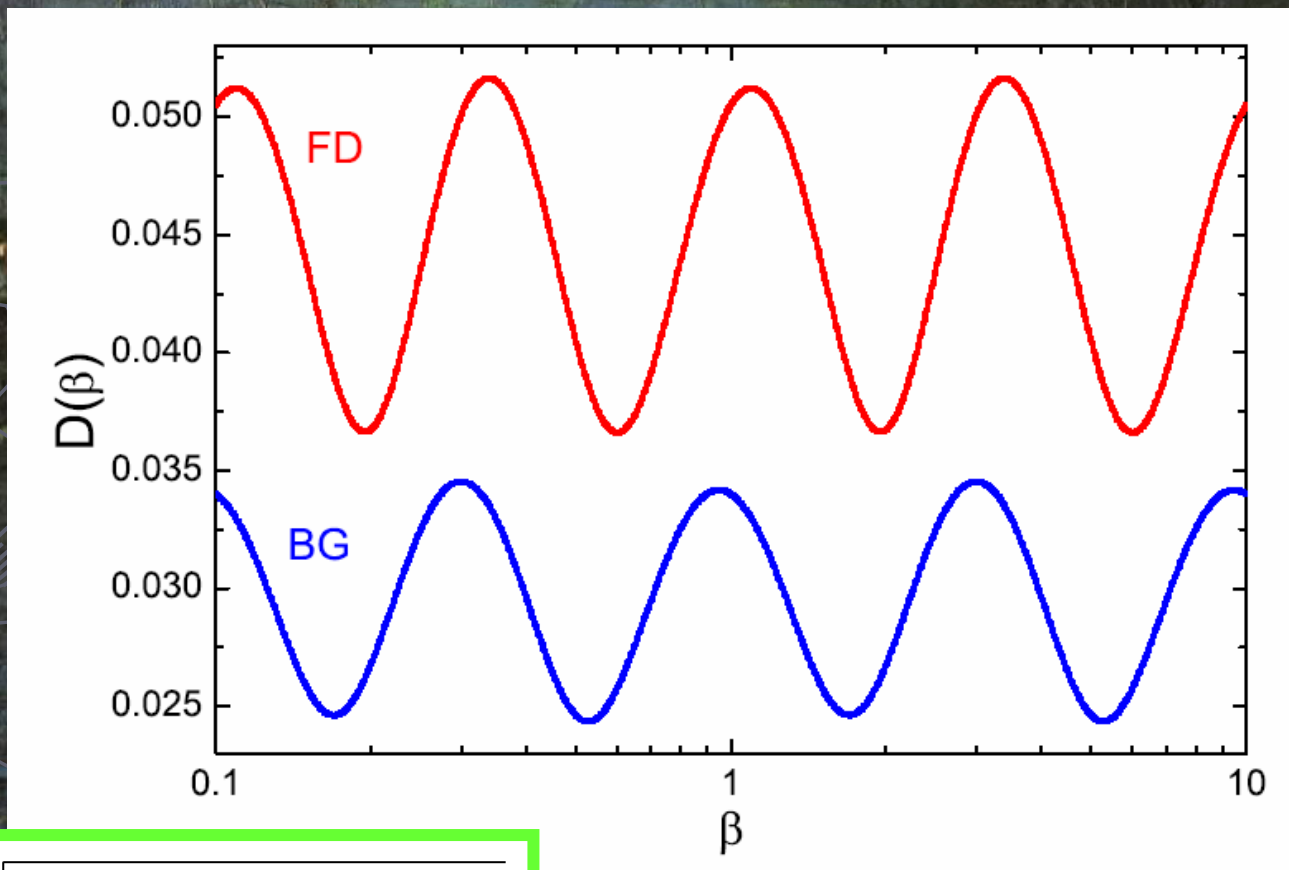
L. Shao & B.-Q. Ma, Physica A (2010) in press

BG与FD分布对Benford定理的最大偏离



L. Shao & B.-Q. Ma, Physica A (2010) in press

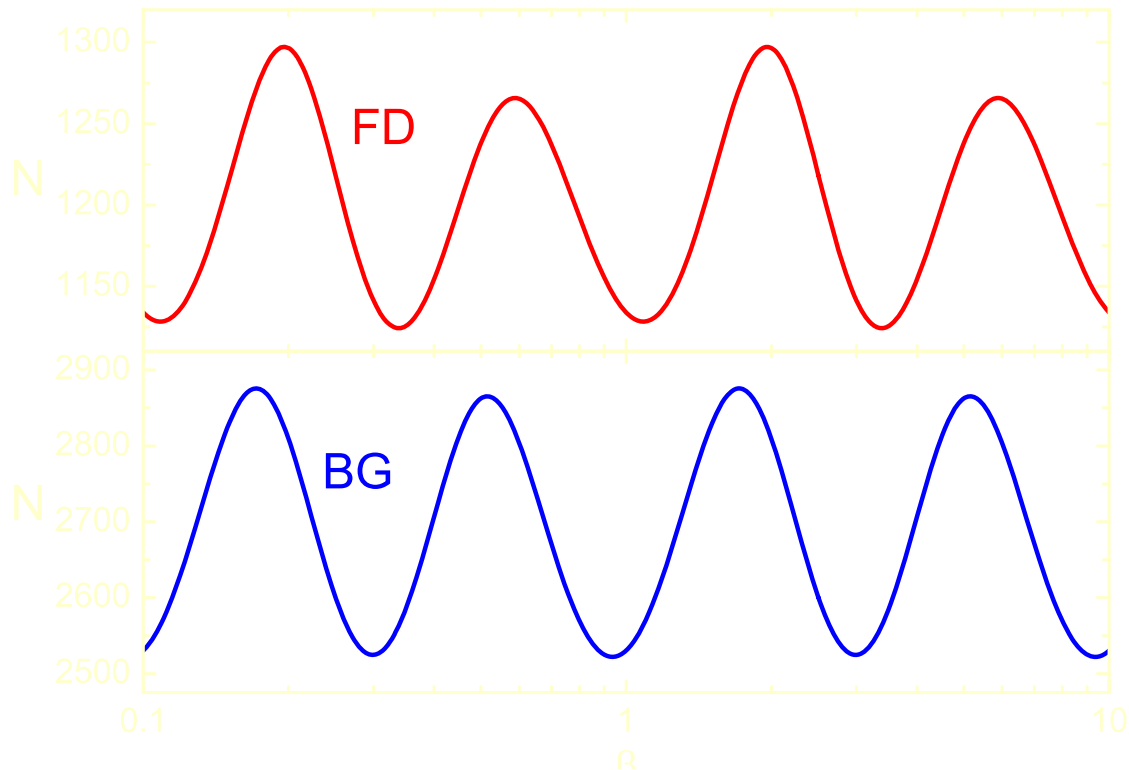
BG与FD分布对Benford定理的偏离



$$D(\beta) = \sqrt{\sum_{d=1}^9 [P(d; \beta) - P_{\text{Ben}}(d)]^2}$$

Shao & Ma, Physica A (2010) in press

能在CL95%下区分物理分布与 Benford分布所需的最小样本容量



L. Shao & B.-Q. Ma, Physica A (2010) in press

化学势 μ 的影响

- ▶ 对玻尔兹曼分布无影响(归一化常数)
- ▶ 对费米分布与玻色分布, 所有相关结论都仍旧成立!

$$f(E) \propto \frac{1}{e^{\beta E} \pm z}$$

$$P(d; \beta, z) = \sum_{n=-\infty}^{\infty} \frac{1}{\ln(1 \pm z)} \left\{ \beta \cdot 10^n + \ln \left[\frac{e^{\beta d \cdot 10^n} \pm z}{e^{\beta(d+1) \cdot 10^n} \pm z} \right] \right\}$$

L. Shao & B.-Q. Ma, Physica A (2010) in press

几点讨论

- ▶ 由于现实中温度的涨落，更会抹平物理分布与Benford分布之间的差别
- ▶ Raimi指出：“各个数据集的和能够更好地符合Benford分布”
 - ▶ Raimi (1969) Sci. Am.
- ▶ Hill指出，如果数据是“随机地”从各种“随机的分布”中取得，那么，它们就会符合Benford定理
 - ▶ Hill (1995) Stat. Sci.
- ▶ 标度不变性： βE 的联合出现

L. Shao & B.-Q. Ma, Physica A (2010) in press

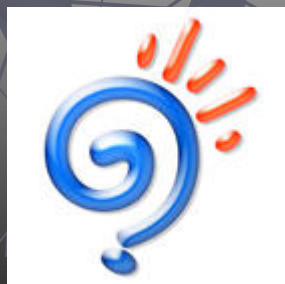
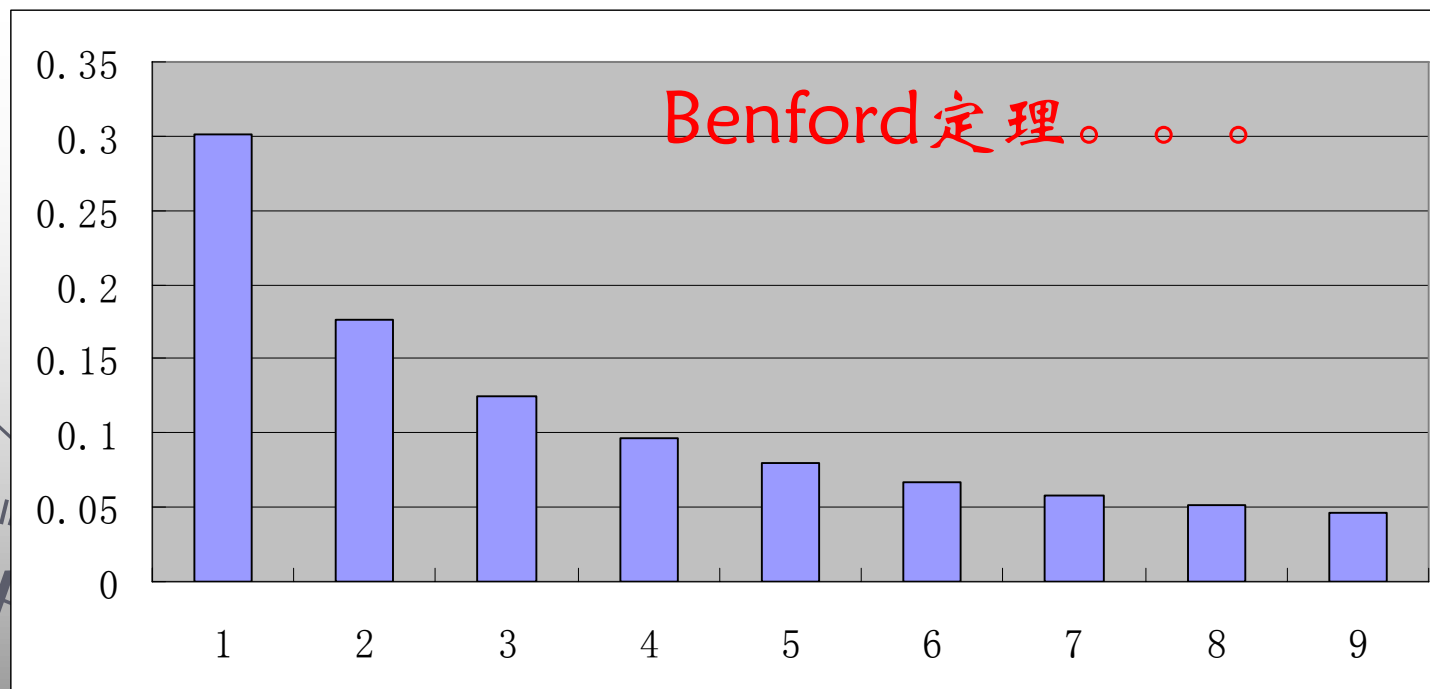
结论 (一)


- ▶ Benford定理自发现100多年来, 还尚没能从理论上完全理解深层的蕴含;
- ▶ 我们首次发现Benford定理也适用于粒子物理领域的强子宽度;
- ▶ 我们首次发现Benford定理也适用于天体物理领域的脉冲星的各种物理量, 并指出, 脉冲星数据库可作为研究Benford定理的理想场所;

结论 (二)

- ▶ 我们首次指出，Benford定理与统计物理中的三大统计定理有出人意料的内在联系。
- ▶ 疑难：如何先验地确定某个数据集是否满足Benford分布？如何理解尾数 $1/m$ 分布？

100多年来的谜团...





The Other Part
科研心得交流



转专业生的我

- ▶ 信息科学技术学院
电子专业 (2005-2007年)
→ 物理学院 (2007年至今)
- ▶ 经济双学位 (hobbies)
- ▶ 天文?



科研经历 (一)

- ▶ 大二上，开始旁听马伯强老师组会
- ▶ 死皮赖脸听不懂。。。。
- ▶ 大二下，申请“君政”本科生科研基金，指导老师：马伯强教授
- ▶ 暑假，强子结构的统计模型
- ▶ PLB09: Zhang, Shao & Ma (真费了九牛二虎之力!)
- ▶ 大三上，在马伯强老师指导下，开始着手做Benford定理
- ▶ MPLA09: Shao & Ma (20天! 投稿辛苦之极!)
- ▶ 大三下，与张运华师兄合作，EMC效应
- ▶ NPA09: Zhang, Shao & Ma

科研经历 (二)

- ▶ 暑假，台湾清华大学，与Albert Kong, Ji-Feng Liu, Yi-Jung Yang合作，做天文数据分析
- ▶ Shao, Liu, Yang & Kong, in preparation
- ▶ 另外，写了两篇Benford定理方面的文章（投稿过程漫长辛苦。。。）
- ▶ ApP10: Shao & Ma
- ▶ PA10: Shao & Ma (3天!)

科研经历 (三)

- ▶ 大四上：在高能所的“第五届夸克物理与核物理国际会议”上做报告
(战战兢兢，如履薄冰)
- ▶ CPC (QNP09) : Shao, Zhang & Ma
- ▶ 与张永军师兄合作，强子结构
- ▶ PLB10: Shao, Zhang & Ma
- ▶ 与肖智师兄合作，做Lorentz破坏
- ▶ ApP10: Shao, Xiao & Ma
- ▶ 大四下，Lorentz破坏 cont.
- ▶ Shao & Ma, submitted

科研方法交流



- ▶ 马伯强老师的指导!
- ▶ 兴趣：原始的好奇心
- ▶ 讨论：组会，讲座，研讨会，学术报告会...
- ▶ 合作者
- ▶ 阅读与视野（对课题的最前沿的了解，对“物理”的理解）
- ▶ 物理思想：Where is the PHYSICS!
- ▶ 努力，很努力
- ▶ **Enjoy it!**

北京大学物理学院“萃英”研究生学术论坛



感謝

感谢马伯强教授的
指导与帮助!

请大家多多指教!

lshao@pku.edu.cn

backup

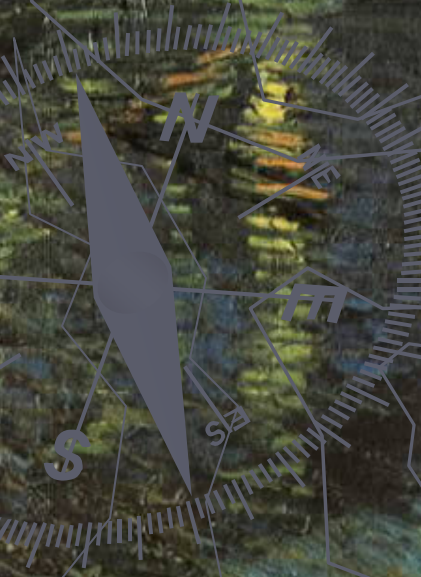


Table 1. The first digit distribution of the full widths of mesons.

| First digit | Case 1 (88) | | Case 2 (91) | | Case 3 (96) | |
|------------------|-------------|------------------|-------------|------------------|-------------|------------------|
| 1 | 24 | (26.5 ± 4.3) | 25 | (27.4 ± 4.4) | 25 | (28.9 ± 4.5) |
| 2 | 22 | (15.5 ± 3.6) | 22 | (16.0 ± 3.6) | 22 | (16.9 ± 3.7) |
| 3 | 11 | (11.0 ± 3.1) | 11 | (11.4 ± 3.2) | 12 | (12.0 ± 3.2) |
| 4 | 9 | (8.5 ± 2.8) | 11 | (8.8 ± 2.8) | 12 | (9.3 ± 2.9) |
| 5 | 5 | (7.0 ± 2.5) | 5 | (7.2 ± 2.6) | 7 | (7.6 ± 2.6) |
| 6 | 2 | (5.9 ± 2.3) | 2 | (6.1 ± 2.4) | 2 | (6.4 ± 2.4) |
| 7 | 5 | (5.1 ± 2.2) | 5 | (5.3 ± 2.2) | 5 | (5.6 ± 2.3) |
| 8 | 6 | (4.5 ± 2.1) | 6 | (4.7 ± 2.1) | 6 | (4.9 ± 2.2) |
| 9 | 4 | (4.0 ± 2.0) | 4 | (4.2 ± 2.0) | 5 | (4.4 ± 2.0) |
| Pearson χ^2 | 6.62 | | 6.82 | | 6.32 | |

Table 2. The first digit distribution of the full widths of baryons.

| First digit | Case 1 (65) | Case 2 (72) | Case 3 (81) |
|------------------|-----------------|-----------------|-----------------|
| 1 | 21 (19.6 ± 3.7) | 22 (21.7 ± 3.9) | 23 (24.4 ± 4.1) |
| 2 | 11 (11.4 ± 3.1) | 12 (12.7 ± 3.2) | 13 (14.3 ± 3.4) |
| 3 | 9 (8.1 ± 2.7) | 11 (9.0 ± 2.8) | 14 (10.1 ± 3.0) |
| 4 | 6 (6.3 ± 2.4) | 6 (7.0 ± 2.5) | 6 (7.8 ± 2.7) |
| 5 | 6 (5.1 ± 2.2) | 7 (5.7 ± 2.3) | 8 (6.4 ± 2.4) |
| 6 | 4 (4.4 ± 2.0) | 5 (4.8 ± 2.1) | 6 (5.4 ± 2.2) |
| 7 | 1 (3.8 ± 1.9) | 1 (4.2 ± 2.0) | 2 (4.7 ± 2.1) |
| 8 | 4 (3.3 ± 1.8) | 4 (3.7 ± 1.9) | 4 (4.1 ± 2.0) |
| 9 | 3 (3.0 ± 1.7) | 4 (3.3 ± 1.8) | 5 (3.7 ± 1.9) |
| Pearson χ^2 | 2.57 | 3.52 | 4.57 |

Table 3. The first digit distribution of the full widths of hadrons.

| First digit | Case 1 (153) | Case 2 (163) | Case 3 (177) |
|------------------|-----------------|-----------------|-----------------|
| 1 | 45 (46.1 ± 5.7) | 47 (49.1 ± 5.9) | 48 (53.3 ± 6.1) |
| 2 | 33 (26.9 ± 4.7) | 34 (28.7 ± 4.9) | 35 (31.2 ± 5.1) |
| 3 | 20 (19.1 ± 4.1) | 22 (20.4 ± 4.2) | 26 (22.1 ± 4.4) |
| 4 | 15 (14.8 ± 3.7) | 17 (15.8 ± 3.8) | 18 (17.2 ± 3.9) |
| 5 | 11 (12.1 ± 3.3) | 12 (12.9 ± 3.4) | 15 (14.0 ± 3.6) |
| 6 | 6 (10.2 ± 3.1) | 7 (10.9 ± 3.2) | 8 (11.8 ± 3.3) |
| 7 | 6 (8.9 ± 2.9) | 6 (9.5 ± 3.0) | 7 (10.3 ± 3.1) |
| 8 | 10 (7.8 ± 2.7) | 10 (8.3 ± 2.8) | 10 (9.1 ± 2.9) |
| 9 | 7 (7.0 ± 2.6) | 8 (7.5 ± 2.7) | 10 (8.1 ± 2.8) |
| Pearson χ^2 | 4.82 | 4.39 | 4.62 |

Table 1

The first digit distributions of the period, frequency, spin down ages, and their time derivatives of pulsars.

| Physical Quantity | Notation | Data points | $\chi^2(8)$ | p -value |
|--|----------|-------------|-------------|------------|
| Barycentric period of the pulsar (s) | P0 | 1825 | 50.552 | 0.0001 |
| Barycentric rotation frequency (Hz) | F0 | 1825 | 54.577 | 0.0001 |
| Time derivative of barycentric period (dimensionless) | P1 | 1695 | 4.497 | 0.8097 |
| Period derivative corrected for proper motion effect | P1_i | 219 | 9.502 | 0.3017 |
| Time derivative of barycentric rotation frequency (s^{-2}) | F1 | 1695 | 7.539 | 0.4797 |
| Second time derivative of barycentric rotation frequency (s^{-3}) | F2 | 395 | 3.020 | 0.9331 |
| Spin down age (yr) [$\tau = P/(2\dot{P})$] | Age | 1664 | 3.721 | 0.8814 |
| Spin down age from P1_i (yr) | Age_i | 219 | 7.078 | 0.5282 |

Table 2

The first digit distributions of the power quantities of pulsars.

| Physical Quantity | Notation | Data points | $\chi^2(8)$ | p -value |
|---|----------|-------------|-------------|------------|
| Spin down energy loss rate (ergs/s) | Edot | 1664 | 6.601 | 0.5802 |
| Energy flux at the Sun (ergs/kpc ² /s) | Edotd2 | 1656 | 9.938 | 0.2694 |
| Radio luminosity at 400 MHz (mJy kpc ²) | R_Lum | 663 | 10.083 | 0.2592 |
| Radio luminosity at 1400 MHz (mJy kpc ²) | R_Lum14 | 1391 | 2.673 | 0.9532 |
| Mean flux density at 400 MHz (mJy) | S400 | 663 | 10.446 | 0.2351 |
| Mean flux density at 1400 MHz (mJy) | S1400 | 1391 | 9.855 | 0.2754 |

Table 3

The first digit distributions of the kinematic quantities of pulsars.

| Physical Quantity | Notation | Data points | $\chi^2(8)$ | p -value |
|---|----------|-------------|-------------|------------|
| Proper motion in declination (mas/yr) | PMDec | 178 | 4.840 | 0.7745 |
| Proper motion in right ascension (mas/yr) | PMRA | 178 | 8.421 | 0.3935 |
| Proper motion in ecliptic latitude (mas/yr) | PMElat | 39 | 13.057 | 0.1099 |
| Proper motion in ecliptic longitude (mas/yr) | PMELong | 83 | 11.695 | 0.1653 |
| Total proper motion (mas/yr) | PMTot | 220 | 7.527 | 0.4810 |
| Transverse velocity - based on DIST (km/s) | VTrans | 219 | 11.855 | 0.1578 |

Table 4

The first digit distributions of the expected minimum companion mass and median companion mass of pulsars.

| Physical Quantity | Notation | Data points | $\chi^2(8)$ | p -value |
|--|----------|-------------|-------------|------------|
| Minimum companion mass assuming $i=90$ degrees and neutron star mass is $1.35 M_{\odot}$ | MinMass | 140 | 31.331 | 0.0001 |
| Median companion mass assuming $i=60$ degrees | MedMass | 140 | 32.781 | 0.0001 |

L. Shao & B.-O. Ma, *Astropart. Phys.* 33 (2010) 255

Table 1: The maximum deviations of the first digit distributions from Benford's law from the BG distribution and the FD distribution.

| d | $P_{\text{Ben}}(d)$ | Max $ \Delta P(d) $ | | Max $ \Delta P(d)/P_{\text{Ben}}(d) $ | |
|-------------|---------------------|---------------------|--------|---------------------------------------|-------|
| First Digit | Benford | BG | FD | BG | FD |
| 1 | 0.301 | 0.0291 | 0.0435 | 9.66% | 14.5% |
| 2 | 0.176 | 0.0187 | 0.0281 | 10.6% | 15.9% |
| 3 | 0.125 | 0.0136 | 0.0204 | 10.9% | 16.3% |
| 4 | 0.097 | 0.0107 | 0.0160 | 11.0% | 16.5% |
| 5 | 0.079 | 0.0088 | 0.0131 | 11.1% | 16.6% |
| 6 | 0.067 | 0.0074 | 0.0111 | 11.1% | 16.6% |
| 7 | 0.058 | 0.0064 | 0.0097 | 11.1% | 16.7% |
| 8 | 0.051 | 0.0057 | 0.0085 | 11.1% | 16.7% |
| 9 | 0.046 | 0.0051 | 0.0076 | 11.1% | 16.7% |